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Peculiarities of Thermoelectric Cooling in p–n Structures

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Abstract The basic physical peculiarities of the phenomenon of thermoelectric cooling in p–n structures by an electric current are investigated in the linear approach. It is proved that, for an adequate research of such structures in the general case, a multi-temperature approximation is required. The multi-temperature approximation means that each of the quasi-particle subsystems (electrons, holes, phonons) is characterized by its own temperature. Also, a criterion is found for applying the one-temperature approximation. The most interesting particular cases are studied, explaining the conditions for each case.

Keywords Temperatures of quasi-particles · Thermoelectric cooling · Thermal transport

1 Introduction

The one-temperature approximation consists of assuming that the temperatures of all quasi-particles (electrons, holes, phonons) in each spatial point coincide [1–5]. Till now, this approximation has been widely used for research of the thermoelectric cool-

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ing phenomenon. However, the criteria for applicability of such a approximation are usually not discussed. Nevertheless, there are many phenomena in which the temperatures of current carriers and phonons are different from each other. The theory of hot electrons [6], when a mismatch of temperatures is connected with Joule heating, is one of such phenomena. Besides heating by an electric current [7], the mismatch of carrier and phonon temperatures can be set by means of a heater and a refrigerator [8]. In this last case the temperatures of charge carriers and phonons are essentially different near the borders of the semiconductor, that is, near the surfaces. They are equal for distances of the order of the cooling length [6,9] from the surfaces. The cooling length is of the order of submicrons [6]. In a study of processes of heat conductivity in [10], it has been shown that the surface is a source of a mismatch of electron and phonon temperatures.

As is well known [11], the Peltier effect, which underlies thermoelectric cooling, is a contact phenomenon. It occurs in a subsystem of electrical charged quasi-particles (electrons, holes). That is, the electron and (or) hole cooling takes place near an interface. Then, the lattice is also cooled by means of energy interactions between electrons (holes) with phonons near the interface. As a result of this two-step process, different temperatures are set in the subsystems of current carriers and phonons.

Thus, it is obvious that the multi-temperature approximation, when each subsystem of quasi-particles is characterized by its own temperature, is more appropriate for the investigation of the thermoelectric cooling phenomenon. However, this problem has not yet been studied.

Traditionally, a p–n structure [12] has been used for the development of thermoelectric refrigerators. Thermal drift fluxes [11] flow in the opposite direction from the interface between both layers of the structure (depending on the current direction), which strengthens the cooling phenomenon. The thermal generation and recombination of electron-hole pairs [12] must take place near the interface for the electric current to take place. As a consequence of the latter fact, non-equilibrium current carriers [13] arise. However, while studying the Peltier effect, nothing has been said so far about these non-equilibrium carriers.

For simplicity we will limit ourselves to the case when recombination processes on the interface are very strong. Then non-equilibrium current carriers are absent, and both electron and hole concentrations are identical at every point of the volume of the corresponding layer (and are equal to their equilibrium concentration) [13]. Actually, in order for this circumstance to take place, the diffusion length of minority carries l_{diff} must be significantly less than both the cooling length l_{c} ($l_{\mathrm{diff}} \ll l_{\mathrm{c}}$) and the lengths $d_{\mathrm{e,h}}$ of the layers of the p–n structure ($l_{\mathrm{diff}} \ll d_{\mathrm{e,h}}$). The cooling length is of the order of 10^{-3}cm to 10^{-4}cm [6]. Diffusion lengths of materials at 300 °C are presented in Table 1 [14].

We are interested in non-degenerated semiconductors. As we can see from Table 1, for some semiconductors the condition $l_{\mathrm{diff}} \ll l_{\mathrm{c}}$ is correct, whereas for others, it is not correct. The theory of thermoelectric cooling for the case $l_{\mathrm{diff}} \geqslant l_{\mathrm{c}}$ was presented in another study [15]. In this current work we want to investigate the influence of finite electron–phonon energy interactions (two-temperature approximation) on thermoelectric cooling in the clearest view. That is why we will investigate the simplest case when the recombination velocity on the interface is sufficiently large ($l_{\mathrm{diff}} \ll l_{\mathrm{c}}$).

Table 1 Diffusion lengths for various materials

Materials	l_{diff} (10^{-3}cm)
n-Si at 10^{17}cm^{-3} donor density	8
p-Si at 10^{17}cm^{-3} acceptor density	10
n-Si at 10^{18}cm^{-3} donor density	2
p-Si at 10^{18}cm^{-3} acceptor density	4
n-Si at 10^{19}cm^{-3} donor density	0.2
p-Si at 10^{19}cm^{-3} acceptor density	0.3
Pure n-Ge	$\geq$200
Pure p-Ge	$\geq$300
Pure n-GaAs ($n_o \sim 10^{14}$cm^{-3})	0.03 to 0.05
Pure p-GaAs (low injection level)	$\sim$0.01
Pure p-GaAs (high injection level)	$\sim$0.07
n-GaN at 10^{16}cm^{-3} electron concentration	0.003
n-GaN at 10^{17}cm^{-3} electron concentration	0.0017
n-GaP at 10^{17}cm^{-3} donor density	0.8
n-GaP at 10^{18}cm^{-3} donor density	0.8
n-GaP at 10^{19}cm^{-3} donor density	0.01
p-GaP at 10^{17}cm^{-3} acceptor density	0.5
p-GaP at 10^{18}cm^{-3} acceptor density	0.5
p-GaP at 10^{19}cm^{-3} acceptor density	0.07
Pure n-InAs ($n_o = 2\times10^{15}$cm^{-3})	0.01 to 0.02
Pure n-InAs	0.03 to 0.06
Pure n-InP ($n_o \sim 10^{14}$cm^{-3})	$\sim$0.04
Pure n-InP ($p_o \sim 10^{15}$cm^{-3})	$\sim$0.08
p-$Ga_{0.47}In_{0.53}As$ at 10^{17} cm^{-3} hole concentration	0.0025
p-$Ga_{0.47}In_{0.53}As$ at 10^{18}cm^{-3} hole concentration	0.0025
p-$Ga_{0.47}In_{0.53}As$ at 10^{19}cm^{-3} hole concentration	0.0005

The purpose of this work consists of both a theoretical investigation of the two-temperature approximation to thermoelectric cooling and the establishment of criteria for applying the one-temperature approximation.

2 General Equations and Boundary Conditions

Let T_{e} and T_{pe} be the temperatures of electron and phonon subsystems, respectively, in the n-layer ($-d_{\text{e}} \leq x < 0$), T_{h} and T_{ph} be the temperatures of holes and phonons in the p-layer ($0 < x \leq d_{\text{h}}$). The current flows normally to the interface along the positive direction of the x axis. Let d_{e} be the thickness of an n-layer and d_{h} be the thickness of the p-layer (see Fig. 1). We consider the cross-sectional area of the structure to

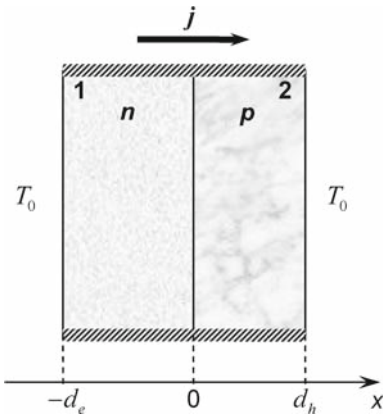

Fig. 1 Schematic diagram of the p–n diode

be equal to unity, and the ambient temperature is T_0. Let us assume that the lateral surfaces of the structure are adiabatically isolated and the materials of the structure layers are homogeneous. In this case, the problem becomes one-dimensional. In the approximation of a linear dependence on the current, the energy balance equations are [6]

$$P_{\mathrm{e}}\left(T_{\mathrm{pe}}-T_{\mathrm{e}}\right)+\kappa_{\mathrm{e}}\frac{\mathrm{d}^2T_{\mathrm{e}}}{\mathrm{d}x^2}=0, \tag{1a}$$

$$-P_{\mathrm{e}}\left(T_{\mathrm{pe}}-T_{\mathrm{e}}\right)+\kappa_{\mathrm{pe}}\frac{\mathrm{d}^2T_{\mathrm{pe}}}{\mathrm{d}x^2}=0, \tag{1b}$$

$$P_{\mathrm{h}}\left(T_{\mathrm{ph}}-T_{\mathrm{h}}\right)+\kappa_{\mathrm{h}}\frac{\mathrm{d}^2T_{\mathrm{h}}}{\mathrm{d}x^2}=0, \tag{1c}$$

$$-P_{\mathrm{h}}\left(T_{\mathrm{ph}}-T_{\mathrm{h}}\right)+\kappa_{\mathrm{ph}}\frac{\mathrm{d}^2T_{\mathrm{ph}}}{\mathrm{d}x^2}=0. \tag{1d}$$

In Eqs. 1a–d, κ_{e} and κ_{pe} are, respectively, the heat conductivities of the electron and phonon subsystems in the n-layer, κ_{h} and κ_{ph} are, respectively, the heat conductivities of hole and phonon subsystems in the p-layer, $P_{\mathrm{e,h}}$ is the coefficient which defines the intensity of the electron–phonon (hole–phonon) energy interaction in the volume. The latter is proportional to the energy frequency of the interaction between electrons (holes) and phonons [6].

In order to correctly formulate the problem in terms of thermoelectric cooling [11], we will assume that there are isothermal contacts at the external surfaces ($x=-d_{\mathrm{e}}$ and $x=d_{\mathrm{h}}$):

$$T_{\mathrm{e,pe}}(-d_{\mathrm{e}})=T_0 \tag{2a}$$

$$T_{\mathrm{h,ph}}(-d_{\mathrm{h}})=T_0 \tag{2b}$$

In traditional investigations of thermoelectric cooling, researchers also use the approximation of isothermal contact for a p–n junction [2,3]. However, the Peltier effect takes

place on this surface, the physics of which is a subject of this investigation. Therefore, it is not correct to simplify the model by making use of an isothermal contact at the p–n transition. We will use the general model, which takes into account all thermal properties of the p–n junction and the possibility of energy interactions at the interface between electrons, holes, and phonons.

Following the procedure outlined in [16], it is not difficult to obtain the following boundary conditions at the interface:

$$j\Pi_s + \eta_{eh}\left(T_e|_{x=-0} - T_h|_{x=+0}\right) - Q_e|_{x=-0} = P_{11}\left(T_{pe}|_{x=-0} - T_e|_{x=-0}\right) + P_{12}\left(T_{ph}|_{x=+0} - T_e|_{x=-0}\right), \quad (3a)$$

$$Q_h|_{x=+0} - \left(j\Pi_s + \eta_{eh}[T_e|_{x=-0} - T_h|_{x=+0}]\right) = P_{21}\left(T_{pe}|_{x=-0} - T_h|_{x=+0}\right) + P_{22}\left(T_{ph}|_{x=+0} - T_h|_{x=+0}\right), \quad (3b)$$

$$\eta_p\left(T_{pe}|_{x=-0} - T_{ph}|_{x=+0}\right) - Q_p|_{x=-0} = P_{11}\left(T_e|_{x=-0} - T_{pe}|_{x=-0}\right) + P_{21}\left(T_h|_{x=+0} - T_{pe}|_{x=-0}\right), \quad (3c)$$

$$Q_p|_{x=+0} - \eta_p\left(T_{pe}|_{x=-0} - T_{ph}|_{x=+0}\right) = P_{12}\left(T_e|_{x=-0} - T_{ph}|_{x=+0}\right) + P_{22}\left(T_h|_{x=+0} - T_{ph}|_{x=+0}\right), \quad (3d)$$

Here,

$$Q_e|_{x=-0} = j\Pi_e - \kappa_e \left.\frac{dT_e}{dx}\right|_{x=-0} \quad (4a)$$

is the electron heat flux at $x = -0$;

$$Q_h|_{x=+0} = j\Pi_h - \kappa_h \left.\frac{dT_h}{dx}\right|_{x=+0} \quad (4b)$$

is the hole heat flux at $x = +0$;

$$Q_p|_{x=-0} = -\kappa_{pe} \left.\frac{dT_{pe}}{dx}\right|_{x=-0} \quad (4c)$$

is the phonon heat flux at $x = -0$; and finally,

$$Q_p|_{x=+0} = -\kappa_{ph} \left.\frac{dT_{ph}}{dx}\right|_{x=+0} \quad (4d)$$

is the phonon heat flux at $x = +0$.

In Eqs. 3 and 4, η_p is the phonon surface heat conductivity, η_{eh} is the electron-hole surface heat conductivity, P_{11} is the coefficient which corresponds to the electron–phonon energy interaction on the interface between the electrons and phonons on the surface of the n-layer of the structure, P_{12} is the coefficient that corresponds to the electron–phonon energy interaction on the interface between the electrons of the n-layer and the phonons of the p-layer, P_{21} is the coefficient that corresponds to the

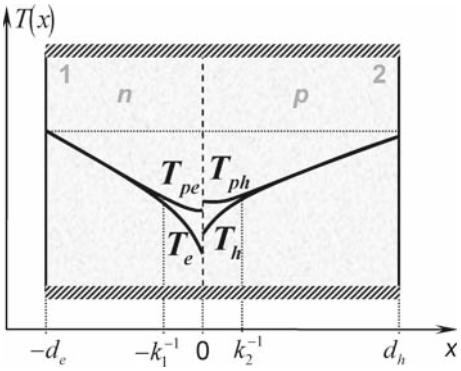

Fig. 2 Schematic diagram of the temperature for thermal cooling in the p–n diode

hole–phonon energy interaction on the interface between the holes of the p-layer and the phonons of the n-layer; P_{22} is the coefficient that corresponds to the hole–phonon energy interaction on the interface between the holes and phonons on the surface of the p-layer of the structure, Π_e is the Peltier coefficient of the n-layer ($\Pi_e < 0$), Π_h is the Peltier coefficient of the p-layer ($\Pi_h > 0$), and Π_s is the Peltier coefficient of the interface [17]. The latter coefficient, according to its definition, can be positive or negative on the p–n junction.

3 Temperature Fields in a p–n Structure

Taking into account the conditions in Eqs. 2 and 3, the solution of Eqs. 1a–d can be written in general as follows (see Fig. 2):

$$T_{e,pe}(x) = T_0 + \frac{1}{D}\left\{G_1\left(1 + \frac{x}{d_e}\right) \pm Q_1 k_{e,pe}^2 \mathrm{sh}(k_1[d_e + x])\right\}, -d_e \leqslant x \leqslant 0 \quad (5a)$$

$$T_{h,ph}(x) = T_0 + \frac{1}{D}\left\{G_2\left(1 - \frac{x}{d_h}\right) \pm Q_2 k_{h,ph}^2 \mathrm{sh}(k_2[d_h - x])\right\}, 0 \leqslant x \leqslant d_h, \quad (5b)$$

Here

$$k_1 = \sqrt{k_e^2 + k_{pe}^2}, \quad (6a)$$

$$k_2 = \sqrt{k_h^2 + k_{ph}^2}, \quad (6b)$$

$$k_{e,h}^2 = \frac{P_{e,h}}{\kappa_{e,h}}, k_{pe,ph}^2 = \frac{P_{e,h}}{\kappa_{pe,ph}}, \quad (7)$$

Since $\kappa_{e,h} \ll \kappa_{pe,ph}$ in non-degenerated semiconductors [6], then $k_1 = k_e, k_2 = k_h$.

The coefficients $D, G_{1,2}$, and $Q_{1,2}$ depend on all parameters of the problem: $\eta_{eh,p}, \kappa_{e,h}, \kappa_{pe,ph}, d_{e,h}, \Pi_{e,h,s}, P_{e,h}, P_{11,12,21,22}$. Expressions for these coefficients are very cumbersome and of little interest in general. Therefore we do not present them here and limit ourselves to a consideration of particular cases.

The temperature distribution that is presented by Eq. 5 is different from those obtained earlier in [11] using the one-temperature approximation, when the energy interaction between electron and phonons is sufficiently large:

$$T_{\mathrm{n,p}}(x) = T_0 \pm \left(\Pi_{\mathrm{n,p}} \pm \frac{\eta d_{\mathrm{p,n}}}{\kappa_{\mathrm{p,n}}} (\Pi_{\mathrm{e}} - \Pi_{\mathrm{h}}) \right) \frac{j(d_{\mathrm{n,p}} \pm x)}{\kappa_{\mathrm{n,p}} \left(1 + \eta \left(\frac{d_{\mathrm{n}}}{\kappa_{\mathrm{n}}} + \frac{d_{\mathrm{p}}}{\kappa_{\mathrm{p}}} \right) \right)}, \tag{8}$$

Here $\Pi_{\mathrm{n,p}}$ are the Peltier's coefficients of semiconductors of n- and p-types, $\kappa_{\mathrm{n,p}}$ are the heat conductivities of semiconductors of n- and p-types, $d_{\mathrm{n,p}}$are the lengths of semiconductors of n- and p-types, respectively, and η is the coefficient of the surface heat conductivity on the interface. The surface Peltier effect is not taken into account in [11].

Comparing Eq. 8 with Eqs. 5a and b, we can see that the basic difference of the two-temperature approximation (see Eq. 5) from that of one-temperature consists of the availability of a term with an exponential temperature distribution which is responsible for the mismatch between the temperatures of electrons and phonons. The basic influence of these terms is near the interface. The temperatures of electrons and phonons are identical at a distance from the interface larger than the cooling length ($k_{\mathrm{e}}^{-1}$and k_{h}^{-1}). However, the linear temperature distribution in Eqs. 5 is also different from the linear temperature distribution in Eq. 8. As a matter of fact, the latter situation takes place because of the exponential temperature distributions near the interface region.

Let us note that the temperature distribution dependence on the surface Peltier coefficient Π_{s} can cause not only cooling, but also heating in the subsystems of quasi-particles (electrons, holes, phonons). The latter depends on the magnitude and the sign of Π_{s}.

We can see from Eq. 5 that the temperature distributions in each subsystem are obtained as the superposition of both the linear term and exponential term as functions of the coordinate x. Furthermore, the linear terms for electrons (holes) and phonons coincide. That is why their difference takes place only at distances smaller than the cooling length (k_{e}^{-1} and k_{h}^{-1}).

Let us consider further those particular cases which are most interesting from a physics point of view.

4 Particular Cases

4.1 Thermally Thin Samples: $k_{\mathrm{e,h}} d_{\mathrm{e,h}} \ll 1$

This case corresponds to the situation when the lengths of layers of the structure $d_{\mathrm{e,h}}$ are significantly less than the corresponding cooling lengths of electrons and holes $k_{\mathrm{e,h}}^{-1} : d_{\mathrm{e,h}} \ll k_{\mathrm{e,h}}^{-1}$.

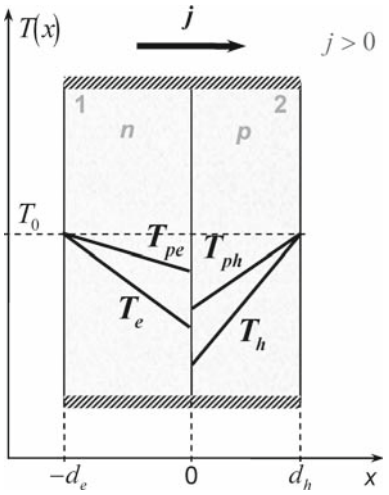

Fig. 3 Schematic diagram of the temperature in the p–n diode for the case when $k_{e,h}d_{e,h} \ll 1$

In this case, the general expressions of Eq. 5 of the temperature distributions simplify to linear dependences on x (see Fig. 3):

$$T_e(x) = T_0 - A_1\,(d_e + x)\,, \tag{9a}$$

$$T_h(x) = T_0 - A_2\,(d_h - x)\,, \tag{9b}$$

$$T_{pe}(x) = T_0 - B_1\,(d_e + x)\,, \tag{9c}$$

$$T_{ph}(x) = T_0 - B_2\,(d_h - x)\,. \tag{9d}$$

Here the coefficients $A_{1,2}$, $B_{1,2}$ depend on all parameters of the problem other than P_e and P_h. We do not present them here because they are cumbersome and of little interest. In Fig. 3 we represent the special case, when cooling of all quasi-particle subsystems takes place.

Let us note that one of the quasi-particle subsystems can be heated, in general. The latter depends on the magnitude and the sign of the surface Peltier coefficient Π_s.

The temperatures become identical (one-temperature approximation) under conditions when $P_{11} \to \infty$ and $P_{22} \to \infty$. But, typically, in this case the interface temperature has a discontinuity (see [11]).

From Eqs. 9a–d it follows that the phonon subsystem is in equilibrium $\left(T_{pe}(x) = T_{ph}(x) = T_0\right)$ if

$$P_{ij} \ll \frac{\kappa_p}{d}, \tag{10}$$

where $P_{ij} \sim P_{11} \sim P_{12} \sim P_{21} \sim P_{22}, d \sim d_e \sim d_h$.

This can be explained by the proportionality of the heat flux magnitude in the phonon subsystem to $\frac{\kappa_p}{d}\left(T_0 - T_p(0)\right)$, where $\kappa_p \sim \kappa_{pe,ph}$, $T_p(0) \sim T_{pe}(-0), T_{ph}(+0)$. Therefore, when d is sufficiently small, then the heat flux in the phonon subsystem

is so large that all energy, which come from the electron (hole) subsystem (which is proportional to $P_{11,12,21,22}$) goes away at once to the thermostat (surroundings) through the surfaces $x = -d_e$ and $x = d_h$.

4.2 Very Thin Samples

$$k_{e,h}d_{e,h} \ll 1;\ \frac{\kappa_e}{d_e}, \frac{\kappa_h}{d_h} \gg \eta_{eh}, \eta_p, P_{ij} \cdot \left(d_{e,h} \ll \frac{\kappa_{e,h}}{\eta_{eh,p}}, \frac{\kappa_{e,h}}{P_{ij}}\right) \tag{11}$$

As follows from the solution, Eq. 8, the temperature distributions in this case are

$$T_e(x) = T_0 + j\frac{\Pi_e - \Pi_s}{\kappa_e}(d_e + x), \tag{12a}$$

$$T_h(x) = T_0 + j\frac{\Pi_s - \Pi_h}{\kappa_h}(d_h - x), \tag{12b}$$

$$T_{pe,ph} = T_0. \tag{12c}$$

We can see from Eqs. 12a, b that the temperature of the electron (hole) subsystem depends only on the electron (hole) parameters and on the magnitude of the surface Peltier coefficient Π_s. The temperature of the electron subsystem decreases when Π_s increases, whereas the hole temperature increases. The electron temperature is equal to the hole temperature at the interface under the condition, $\frac{\kappa_e}{\kappa_h}\frac{d_h}{d_e} = \frac{\Pi_e - \Pi_s}{\Pi_s - \Pi_h}$. This is possible if $\Pi_e < \Pi_s < \Pi_h$.

Using the one-temperature approximation in this case (see [11]), we have the following:

$$T_{n,p}(x) = T_0 \pm j\frac{\Pi_{n,p}}{\kappa_{n,p}}(d_{n,p} \pm x) \tag{13}$$

Comparing Eqs. 12a–c with Eq. 13, we can see that the former result in Eq. 13 is sufficiently different from that in Eqs. 12a–c. From Eqs. 12a–c, we see that the phonon subsystem is in equilibrium, whereas the electron subsystem is cooled much more than that determined from Eq. 13. The latter fact is due to $\kappa_{e,h} \ll \kappa_{pe,ph}$.

From Eqs. 12a–d, we can see that the electron (hole) subsystem is not aware of the existence of the hole (electron) and phonon subsystems. Hence, the phonon subsystem cannot be taken into account (it is in equilibrium) while investigating p–n structures, the layer lengths of which correspond to the condition in Eq. 11. On the other hand, the electron and hole subsystems can be investigated independently of each other. This is connected with the domination of heat flux processes. The latter is proportional to $\frac{\kappa_{e,h,pe,ph}}{d_{e,h}}$, and has the dominating role on forming a temperature distribution in such thin samples. Whereas, in the processes of heat exchange, which are proportional to $\eta_{eh,p}$, $P_{11,12,21,22}$ and $P_{e,h}$ $(k^2_{e,h})$ are not important.

4.3 Thermally Thick Samples: $k_{e,h}d_{e,h} \gg 1$

This case corresponds to the situation when each one of the two lengths of the layer of the p–n structure $d_{e,h}$ is larger than the corresponding cooling lengths of the electrons and holes $k_{e,h}^{-1}$: $d_{e,h} \gg k_{e,h}^{-1}$.

The general distributions of the temperatures of Eq. 5 in this case are as follows (see Fig. 2):

$$T_{e,pe}(x) = T_0 + \frac{1}{L}\left\{C_1\left(1 + \frac{x}{d_e}\right) \pm B_1 k_{e,pe}^2 e^{k_e x}\right\}, \tag{14a}$$

$$T_{h,ph}(x) = T_0 + \frac{1}{L}\left\{C_2\left(1 - \frac{x}{d_h}\right) \pm B_2 k_{h,ph}^2 e^{-k_h x}\right\}, \tag{14b}$$

Here L, $C_{1,2}$ ($C_{1,2} < 0$), and $B_{1,2}$ ($B_{1,2} > 0$) are the coefficients, which depend on all parameters of the problem. We can see from these expressions that the linear distributions are identical for the electron (hole) and phonon subsystems. The difference of these temperatures is due to the exponential terms of the distributions which are important only near the interface at distances of the cooling length $k_{e,h}^{-1}$. The cooling case is represented in Fig. 2, but we should be aware that, in principle, heating can also take place instead of cooling; which process is going to take place depends on the sign and magnitude of Π_s.

As we can see from Eqs. 14a and b, contrary to the linear temperature distribution in the one-temperature approximation (see Eq. 8 [11]), the temperatures undergo a mismatch near the interface; the electron temperature undergoes additional cooling, and the phonon temperature cooling is less than in the one-temperature approximation.

In order to understand better the dependence of the temperature distributions on the parameters of the problem, let us examine our results. For a simple evaluation, we assume:

$$\eta_e \sim \eta_p \sim P_{11} \sim P_{12} \sim P_{21} \sim P_{22} \sim \eta, \tag{15a}$$

$$\kappa_e \sim \kappa_h \sim \kappa_{eh}, \tag{15b}$$

$$\kappa_{pe} \sim \kappa_{ph} \sim \kappa_p, \tag{15c}$$

$$P_e \sim P_h \sim P \tag{15d}$$

$$d_e \sim d_h \sim d. \tag{15e}$$

It follows from Eqs. 15b–d, 6, and 7 that

$$k_e \sim k_h \sim k. \tag{16}$$

4.4 Very Thick Samples

A physically interesting result arises when the exponential parts of the distributions are not significant compared to the linear parts. Analyzing Eq. 14, we can see that this

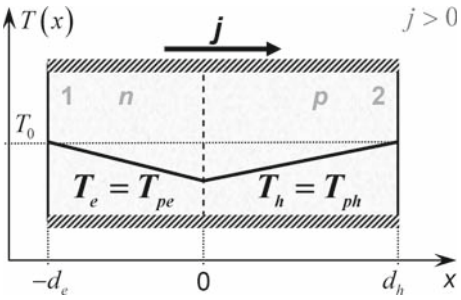

Fig. 4 Schematic diagram of the temperature in the p–n diode under the condition of Eq. 17

situation takes place if

$$d \gg \frac{1}{k}, \frac{\kappa_{\mathrm{p}}}{\eta + k\kappa_{\mathrm{eh}}}, \frac{\kappa_{\mathrm{p}}}{\eta} \frac{\left|\Pi_{\mathrm{h,e}} - \Pi_{\mathrm{s}}\right|}{\Pi_{\mathrm{h}} - \Pi_{\mathrm{e}}}. \tag{17}$$

At these circumstances, the temperatures are (see Fig. 4)

$$T_{\mathrm{e,pe}}(x) = T_0 - \frac{j\left(\Pi_{\mathrm{h}} - \Pi_{\mathrm{e}}\right)}{\kappa_{\mathrm{p}}}(d + x), \tag{18a}$$

$$T_{\mathrm{h,ph}}(x) = T_0 - \frac{j\left(\Pi_{\mathrm{h}} - \Pi_{\mathrm{e}}\right)}{\kappa_{\mathrm{p}}}(d - x). \tag{18b}$$

We can see from these distributions that the electron (hole) temperature coincides with the phonon temperature and also that there is no discontinuity at the interface. Moreover, we see that these distributions do not depend on the surface Peltier coefficient Π_{s}. Hence, the surface Peltier coefficient influences the temperature distribution only in the case of not very thick samples. Besides, since Eq. 18 does not depend on the electron and hole heat conductivities, it means that the drift heat fluxes [11] have the dominating role whereas diffusion heat fluxes of the electrons and holes are not essential [7]. Hence, for very thick samples, as seen from Eq. 18, the temperature distribution in the p–n structure of non-degenerated semiconductors depends, under conditions in Eq. 17, only on the parameters κ_{p}, $\Pi_{\mathrm{e,h}}$, and $d_{\mathrm{e,h}}$.

Thus, for sufficiently thick samples, it is quite correct to use the one-temperature approximation with isothermal contact on an interface (see Eq. 17). Normally, it is precisely this case that is studied by researchers when one wants to investigate the cooling phenomenon in p–n structures. However, the conditions for using this approximation are never discussed [1–5]. As follows from Eq. 8, the results are identical for very thick samples (see Eq. 17) in the one-temperature and two-temperature approximations.

If the energy interaction at the interface between charge carriers (electrons, holes) and phonons is much smaller than the surface heat conductivity $\eta_{\mathrm{eh}} \sim \eta_{\mathrm{p}} \sim \eta \gg P_{ij}$, instead of the criterion in Eq. 17, we obtain another result:

$$d \gg \frac{\kappa_{\mathrm{p}}}{k\kappa_{\mathrm{e}}}, \frac{\kappa_{\mathrm{p}}}{\eta + k\kappa_{\mathrm{e}}} \frac{\left|\Pi_{\mathrm{e,h}} - \Pi_{\mathrm{s}}\right|}{\Pi_{\mathrm{h}} - \Pi_{\mathrm{e}}}. \tag{19}$$

In this last situation the temperature distributions are the same (see Eqs. 18a, b).

Note that there can be the case when the linear part of the temperature distribution is not essential compared with the exponential part. The difference of the temperatures will be localized only near the interface.

5 Conclusions

We studied the details of thermoelectric cooling in p–n structures of any lengths in the two-temperature approximation. We were interested in the general case when the interactions between all the quasi-particles (electrons, holes, phonons) take place, both in the volume and at the p–n interface. It is shown that, for the investigation of thermal cooling in p–n structures, the multi-temperature approximation (when every subsystem of quasi-particles is characterized by its own temperature) must be used. Cooling or heating can take place in a p–n structure when a current passes from an n-layer to a p-layer; which case occurs depends on both the sign and the magnitude of the surface Peltier coefficient. We determined how thin a p–n structure must be for cooling only the charged particles, but not the phonons.

We show that, for thermally thick samples, the temperatures of the electrons (holes) can be different from the phonon temperature only at distances of the cooling length near the interface. We obtained criteria for the sizes of the layers of the p–n structure in order to use the one-temperature approximation with isothermal contact at the p–n interface (see Eq. 17). It is shown that, in this case, the temperature distribution does not depend on the surface Peltier coefficient. This fact means that only cooling of the p–n structure takes place.

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